Written test, May 25, 2009

Course: Teletraffic Engineering & Network Planning Course no. 34 340

Aids allowed: All

Weighting: All questions have the same weight. The evaluation is complemented with an overall appraisal of the solution

Exercise 1

Priority queueing system

We consider a single server queueing system with two types of customers arriving according to Poisson processes.

- Customers of type one has arrival rate $\lambda_1 = 0.2$ [customer/time unit].
  The service time is constant with mean value $m_{1,1} = 1$ [time unit].
- Customers of type two has arrival rate $\lambda_2 = 0.2$ [customer/time unit].
  The service times are hyper-exponentially distributed with two branches (H_2):
  90 % of the customers have mean service time $m_{1,a} = 1$ [time unit],
  10 % of the customers have mean service time $m_{1,b} = 21$ [time units].

1. Show that the service time distribution of type two customers has mean value $m_{1,2} = 3$ [time units], second moment $m_{2,2} = 90$ [time units^2], and form factor $\varepsilon = 10$.
2. Find the offered traffic for each type of customer, and the total offered traffic.
3. Find by using Pollaczek-Khintchine's formula the mean waiting time W for all customers. Also find the mean waiting time w for delayed customers.

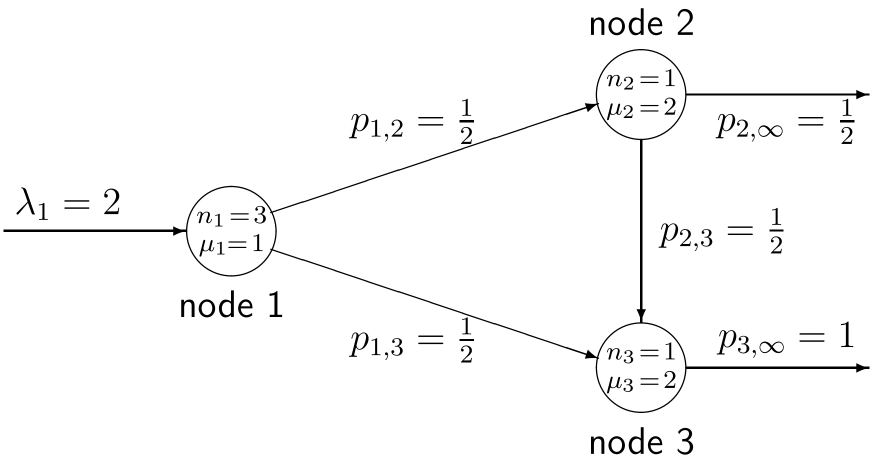
We now assume that type one customers have higher priority than type two customers.

4. Assume non-preemptive priority and find the mean waiting time for each type of customers.
5. Assume preemptive-resume priority and find the mean waiting time for each type of customers.

Exercise 2

Queueing network with three nodes

We consider an open queueing network with three nodes as shown in the figure.



- Node one is an *M/M/3* queueing system with mean service time $s_1 = 1$ [time unit]. Calls arrive from outside to node one according to a Poisson process with rate $\lambda_1 = 2$ [customers/time unit]. From node one the routing probability is $p_{1,2} = 1/2$ to node two, and $p_{1,3} = 1/2$ to node three.
- Node two is an *M/M/1* queueing system with mean service time $s_2 = 1/2$ [time units]. From node two the routing probability is $p_{2,3} = 1/2$ to node three, and with probability $p_{2,\infty} = 1/2$ a customer leave the network.
- Node three is an *M/M/1* queueing system with mean service time $s_3 = 1/2$ [time units]. From node three customers leave the network ($p_{3,\infty} = 1$).

1. Find the traffic offered to each node.
2. Find the state probabilities $p_i(j)$ for state $j = 0, 1, 2, 3, 4$ for each node ($i = 1, 2, 3$), and the state probability $p(x_1, x_2, x_3) = p(1, 1, 1)$ for the whole queueing network.
3. Find the mean waiting time for all customers in each node.

We now close the network by fixing the total number of customers to 4 customers. Thus we only look at states with a total number of 4 customers. (Customers which leave the network in node 2 and 3 immediately go to node one).

4. Find by convolving the above state probabilities the state probabilities of node three.
5. Find the carried traffic in node three, and then the carried traffic in the other two nodes.

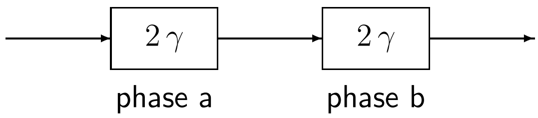
Exercise 3

Engset's loss system and insensitivity to idle times

We consider Engset's loss system with $n = 3$ channels. The traffic is generated by $S = 5$ sources. An idle source generates call attempts with intensity $\gamma = 1$ [call attempt/time unit]. The mean service time is $\mu^{-1} = 1$ [time unit].

1. Find the offered traffic.
2. Construct the state transition diagram and find the state probabilities, time congestion, traffic congestion, and call congestion.

We now want to indicate by an example that the above state probabilities are insensitive to the idle-time distribution. We assume that the idle-time distribution is Erlang-2 distributed (phase a and b) with the same rate $2\,\gamma$ in both phases so that the mean-idle time still is one [time unit] as above. We define the state of the system as (i, j, k), where i is the number of



idle sources in first phase (a), j is the number of idle sources in second phase (b), and k is the number of busy sources (or channels). Note that $i+j+k = 5$ so that the state-transition diagram is only two-dimensional.

3. Fill in the transition rates in the two-dimensional state transition diagram given below.

To be insensitive it can be shown that for a number of busy sources k, corresponding to a row in the state transition diagram, the distribution of the number of idle sources in phase a and b must be Binomial distributed so that

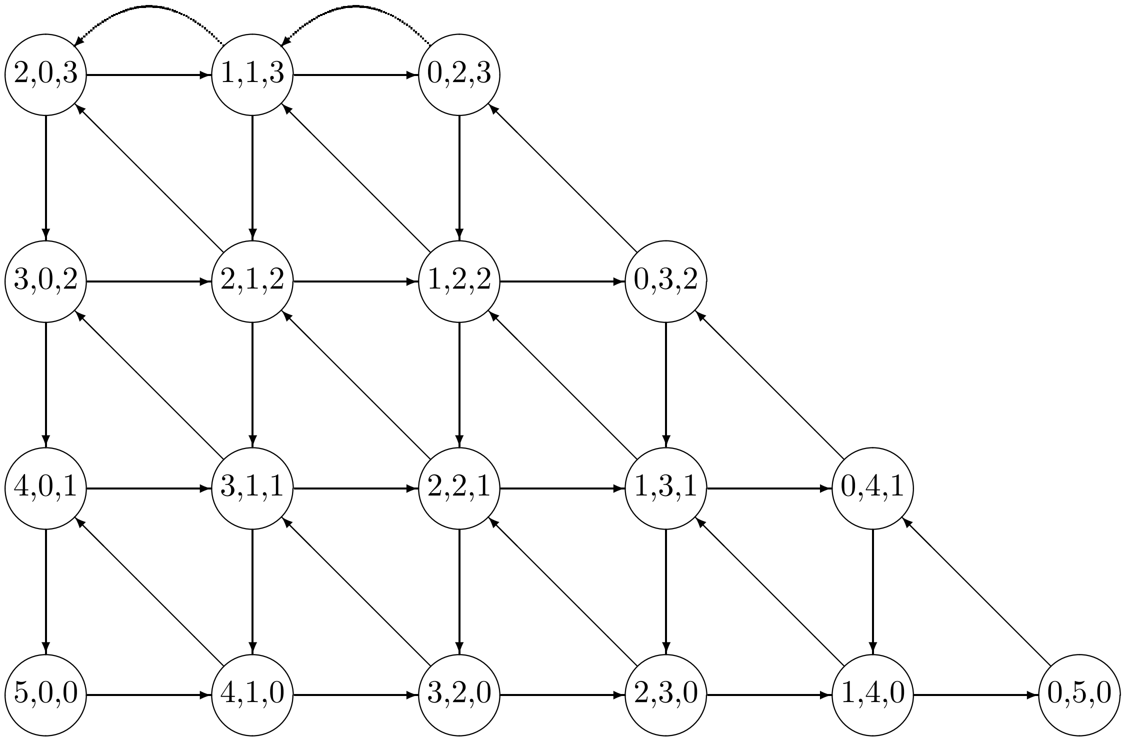
$$p(i, j \mid k) = \binom{5-k}{i} \cdot \left(\frac{m_{1,a}}{m_{1,a} + m_{1,b}}\right)^{i} \cdot \left(\frac{m_{1,b}}{m_{1,a} + m_{1,b}}\right)^{5-k-i} .$$

Inserting the actual values we get:

$$p(i, j, k) = \binom{5-k}{i} \cdot \left(\frac{1}{2}\right)^{5-k} \cdot p(k)\,, \quad k = 0, 1, 2, 3\,,$$

where $p(k)$ are the state probabilities obtained above in Question 2.

4. Find these state probabilities (express for example all state probabilities by the fraction $x/832$, then all values of x becomes integers and $p(0, 0, 0) = 1/832$.
   Show the state probabilities fulfil the node balance equations by considering the node balance equations for state $(1, 2, 2)$.
5. Find an expression for the call congestion B from the two-dimensional state probabilities. (As a control we get the same numerical value as in Question 2, which indicates that the the Engset model is insensitive to the idle time distribution).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3,0,2
2,1,2
1,2,2
0,3,2
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1,3,1
0,4,1
5,0,0
4,1,0
3,2,0
2,3,0
1,4,0
0,5,0

This page can be used for the solution