

A SURVEY OF A. K. ERLANG'S ELECTROTECHNICAL WORKS

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At the time when Erlang began his work in the service of the Copenhagen Telephone Company one of the problems that had special interest for the Company was the use of cables with artificially increased self-induction in order to improve the transmission quality. There were two solutions of this problem to choose between, *viz.* the method of increasing the self-induction of a circuit at uniformly spaced points, as suggested by Prof. *M. Pupin*, and the method of increasing the self-induction continuously, as suggested by *C. E. Krarup*, M. Sc., Chief Engineer of the Danish State Post & Telegraph Administration, and *J. L. W. V. Jensen*, M. Sc., Ph. D., Engineer-in-Chief to the Copenhagen Telephone Company; consequently it became part of Erlang's duties to carry out a great deal of calculations of various kinds, respecting *e. g.* the profitableness of different systems, the optimum interval between Pupin coils, the maximum reduction in attenuation by Krarup's method, ideal loading, &c. Most of these works are purely calculative even though they comprise several theoretical works also. Only one of the latter has been published, *viz.*:

9. *An Elementary Treatise on the Main Points of the Theory of Telephone Cables*, p. 233¹).

Published in Danish under the title of
Hovedpunkterne af Teorien for Telefonkabler i elementær Fremstilling.
Elektroteknikereren, vol. 7, 1911, p. 139.

After a historical, mathematical, and physical introduction to this work, Erlang deduces in a simple manner the principal formulæ concerning the infinitely long, homogeneous cable; the results are then applied especially to cables with artificial, continuously distributed self-induction.

¹) The numbers prefixed in this survey to the titles of Erlang's reprinted works correspond to the numbers of the reprints in the present book, to which also the suffixed page numbers have reference.

It is shown that any length of cable is characterized by 4 principal constants, only 3 of which need be known as all 4 principal constants are linearly interdependent. The relationship existing between the principal constants and some constants mentioned in the foregoing is demonstrated, and rules of calculation are given for the connecting of cables in series. Next, the coil-loaded, or Pupin, cables are mentioned, and the cooperation of the receiving instrument with the line, and finally the application of the theory to the measuring methods.

As an example of Erlang's works on other subjects may be mentioned the following:

10. *An Elementary Theoretical Study of the Induction Coil in a Subscriber's Telephone Apparatus*, p. 253.

First published in Danish:

Transformatoren i et Telefonapparat, en elementær teoretisk Undersøgelse.

Elektroteknikeren, vol. 10, 1914, p. 169.

Later published in French:

Etude théorique élémentaire sur le transformateur d'un appareil téléphonique.

La Vie Technique et industrielle, vol. 9, 1927, octobre.

Having formulated, in this paper, the necessary assumptions, Erlang sets forth the basic equations that give the conditions with which a serviceable induction coil must comply; the theory is then applied to an example.

Being occupied with the theoretical problems presented by telephone cables, Erlang soon found himself wanting an instrument for measuring the transmission constants of cables, and this led to the construction of his "Complex Compensator". This compensator, constituting a decided improvement as compared with complex compensators of earlier date, is described in:

11. *New Alternating-Current Compensation Apparatus for Telephonic Measurements*, p. 261.

Journal of the Institution of Electrical Engineers, vol. 51, 1913, p. 794.

First published in Danish:

Et nyt Kompensationsapparat til Vekselstrømsmaalinger indenfor Telefonien.

Elektroteknikeren, vol. 9, 1913, p. 157.

The apparatus contains two measuring wires, connected in parallel; which are supplied with current through resistors and inductors, respectively. Potentials can be tapped from the two measuring wires by means of sliding-contacts; the amplitude and phase of these potentials can be adjusted so as to compensate an unknown potential. The apparatus can be used for measuring impedances, for transmission measurements on telephone circuits, and for measuring frequencies.

Erlang's complex compensator is easy to set up and easy to operate; but the computation of the results takes some time, especially when measuring on a range of frequencies, as the voltage vectors refer to oblique coordinate axes whose angle varies with the frequency. By altering Erlang's compensator, Prof. *P. O. Pedersen* a few years later succeeded in constructing a compensator¹⁾ with voltage coordinates which remain rectangular for all frequencies. *P. O. Pedersen's* compensator, however, still has a disadvantage in that the scale by which abscissae and ordinates are measured varies with the frequency, though the device may be made direct-reading for a standard frequency; it has been put into practical form by Messrs. *H. Tinsley & Co.*

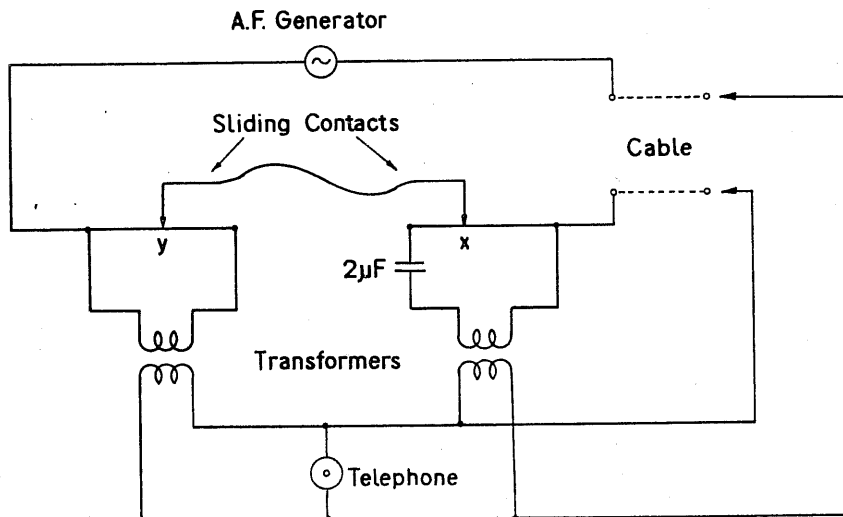
Not satisfied that his problem had been solved with the publication of the above-mentioned paper on the complex compensator, Erlang occasionally took it up for renewed treatment. Thus, among his unpublished works is the construction of a so-called "complex current-compensator", the principle of which will appear from the subjoined diagram showing the current-compensator when used for measuring the attenuation in a cable. A paper by *H. T. Stenby*²⁾ contains a brief description of the apparatus.

The current-compensator contains two air-core transformers, the primaries of which are connected in series and get their supply of current through sliding-contacts as shown in the diagram. Owing to a capacitor being inserted in series with one primary, the secondaries can be tapped for currents, the amplitude and phase of which can be varied so as to compensate an unknown current. On the basis of the constants of the compensator it is possible, for any desired frequency, to work out a diagram arranged in such a manner that the attenuation can be read directly as a function of the positions of the sliding-contacts along the slide-wire.

¹⁾ *P. O. Pedersen*: A New Alternating-Current Potentiometer for Measurements on Telephone Circuits, *Electrician*, vol. 83, 1919, p. 523.

²⁾ *H. T. Stenby*: Nogle Vekselstrømsmaalinger i Telefontekniken, *Ingeniøren*, vol. 43, 1934, II, p. 17.

This attenuation diagram consists of a family of Booth's lemniscates¹⁾, graphed relative to a system of oblique coordinate axes.



In continuation of his experiments with the current-compensator, Erlang had commenced the preliminary work in connexion with a new voltage-compensator of a modified construction, but he did not live to complete it.

At his death Erlang left several notes — especially on Pupin cables, but also on the balancing of two-wire repeater circuits and other problems — which were not intended for publication. The only electrotechnical studies published by Erlang are the three works commented upon above and reprinted in the present book; they will serve to illustrate Erlang's faculty of applying mathematical points of view to the solution of electrotechnical problems in telephony.

¹⁾ These curves, the corresponding Cartesian equation of which is $(x^2 + y^2)^2 = a^2 x^2 \pm b^2 y^2$, are elliptic or hyperbolic according as the sign of the last member on the right side of the equation is + or —. In the latter case, the curves for $a^2 = b^2$ will be ordinary (Bernoulli's) lemniscates, $(x^2 + y^2)^2 = a^2 (x^2 - y^2)$. The curves referred to above have been investigated by *J. Booth* in "A Treatise on some New Geometrical Methods", London, 1877, vol. I, p. 162 ff. The name of "Booth's lemniscates" was given by *G. Loria* (see: *G. Loria*: "Spezielle algebraische und transzendente ebene Kurven", 2. Aufl., 1910, vol. I, p. 134).